

Further Maths Revision Paper 6

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q1 and Q5)

1

Prove by induction that

$$\sum_{r=1}^n \frac{r2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$$

2

Find the Maclaurin expansion, upto and including the term in x^4 of

$$\ln(\cos x)$$

3

(a) Expand $\left(z + \frac{1}{z}\right)^4$

(b) Hence, by considering $\left(z + \frac{1}{z}\right)^4$ and $\left(z - \frac{1}{z}\right)^4$, with $z = \cos \theta + i \sin \theta$ show that

$$\cos^4 \theta + \sin^4 \theta = \frac{1}{4} (\cos 4\theta + 3)$$

4

By means of the substitution $y = vx$ reduce the differential equation

$$xy \frac{dy}{dx} = y^2 + \sqrt{x^2 + y^2}$$

to an equation in v and x .

Find the solution, given that $y = 1$ when $x = 1$ in the form $y^2 = f(x)$

5

Prove that

$$\frac{\sin \theta}{1 - \cos \theta} \equiv \cot \frac{1}{2}\theta$$